Absolute Drift

Year \ No. of firms	Size of Firm £80	£90	£100	£110	£120
0	0	0	128	0	0
1	0	32	64	32	0
2	8	32	48	32	8

Total number of firms = 128

$$CR_{10}^{0} = \frac{1000}{12800} = 0.078$$

$$CR_{10}^{1} = \frac{1100}{12800} = 0.086$$

$$CR_{10}^{2} = \frac{120 \cdot 8 + 110 \cdot 2}{12800} = \frac{118}{1280} = 0.092$$

Relative Drift

Year \ No of firms	Size of firm £25	£50	£100	£200	£400
0	0	0	128	0	0
1	0	32	64	32	0
2	8	32	48	32	8

Total number of firms = 128

$$CR_{10}^{0} = \frac{1000}{12800} = 0.078$$

$$CR_{10}^{1} = \frac{2000}{14400} = 0.138$$

$$CR_{10}^{2} = \frac{8 \cdot 400 + 2 \cdot 200}{16200} = \frac{3600}{16200} = 0.222$$

Gibrat's Law -The Law of Proportionate Effect (LPE)

S_0 →tbe firm size in period 0

S_1 →the firm size in period 1

g_1 →proportional growth rate for period 1

Then

$$S_1 = S_0(1+g_1) \Rightarrow \log(S_1) = \log(S_0) + \log(1+g_1)$$

$$\text{or } X_t = X_{t-1} + e_t \qquad (1)$$

$X_t = \log(S_t)$, t=1,2, ,T,

$e_t = \log(1+g_t)$ is the normally distributed error term with variance s^2

The LPE makes two assumptions

a) there is no correlation between a firm's size and growth between periods

→ $Cov(X_{t-1}, e_t) = 0$

b) no correlation between successive growth rates. → $Cov(e_t, e_{t-s}) = 0$.

Then this equation yields for period 1

$$Var(X_1) = Var(X_0) + Var(e_1) + 2Cov\ (X_0, e_1) \qquad (2)$$

but the last term on the R.H.S. is zero since we have assumed that the growth of the firm is independent of its initial size. Then

$$\sigma_1^2 = \sigma_o^2 + s^2 \text{ with } \sigma_t^2 = Var(X_t),\ s^2 = Var(e_t) \qquad (3)$$

For period 2:

$$\sigma_2^2 = \sigma_1^2 + s^2 = \sigma_0^2 + 2s^2$$

Generalising to period t:

$$\sigma_t^2 = \sigma_{t-1}^2 + s^2 = \sigma_0^2 + ts^2 \qquad (4)$$

In other words the variance of log size will be the product of the variance of growth rate times the number of periods that have elapsed. Clearly from (4):

$$\lim_{t\to\infty} \sigma_t^2 = \infty \qquad (5)$$

ie, the variance (spread) of the firm sizes increases indefinitely over time. This is the testable implication of Gibrat's Law. Note that since the random variable e_t follows the normal distribution with variance s^2, this implies that as $t \to \infty$ X_t (the log of firm size) also follows the normal distribution. Hence the size distribution of firms will have the Log-Normal distribution.

Prais generalises this model by introducing the following:

$$X_t = bX_{t-1} + e_t \Rightarrow \sigma_t^2 = b^2 Var(X_{t-1}) + Var(e_t) = \beta\sigma_{t-1}^2 + s^2 \quad (6)$$

Assume again $Cov(X_{t-1},e_t) = Cov(e_t,e_{t-s}) = 0)$ where e_t is the normally distrubuted random error) and set $\beta = b^2$. Hence

$$\sigma_1^2 = \beta\sigma_0^2 + s^2 \Rightarrow$$

$$\sigma_2^2 = \beta\sigma_1 + s^2 = \beta(\beta\sigma_0^2 + s^2) + s^2 = \beta^2\sigma_0^2 + \beta s^2 + s^2 \Rightarrow$$

$$\sigma_t^2 = \beta^t\sigma_0^2 + s^2(1+\beta+\beta^2+\ldots+\beta^{t-1}) \Rightarrow$$

$$\sigma_t^2 = \beta^t\sigma_0^2 + s^2\left(\frac{1-\beta^t}{1-\beta}\right) \text{ for } 0<\beta<1 \quad (7)$$

Therefore, since if $0<\beta<1$ as $t \to \infty$ $\beta^t \to 0$ we have

$$\lim_{t\to\infty} \sigma_t^2 = \begin{cases} \dfrac{s^2}{1-\beta} & \text{if } 0<\beta<1 \quad (0<b<1) \\ \infty & \text{if } \beta \geq 1 \quad (b \geq 1) \end{cases}$$

In the case of $\beta<1$ we have the Galtonian Regression where small firms will grow proportionately more than large firms. The firm's growth depends on its size since from (6) we may write

$$X_t - X_{t-1} = (b-1)\, X_{t-1} + e_t$$

Then differentiating

$$\frac{\partial(X_t - X_{t-1})}{\partial X_{t-1}} = b-1 < 0$$

In other words, a unit increase in size leads to a b-1 drop in the firm's growth rate over the coming period. (That is, the Gibrat assumption that growth rate is independent of size is violated; we shall see this result later in Weiss.) As a result, the variance of firm sizes does **not** increase indefinitely but tends to a finite limit which depends (positively) on the variance of growth rates (s^2) and the magnitude of b.

Correlation of Growth and Initial Size (Weiss, 1963)

Weiss recognises that the variance of logs of firm sizes as such measures inequality rather than concentration since it ignores firm numbers. Increasing variance implies increasing concentration only if firm numbers do not alter. Define

$$X_t = \log(S_t)$$

where S_t the size of firm in t, t=1,2. Then we can write the identity

$$X_2 = X_1 + (X_2-X_1)$$

where X_2-X_1 is the growth of firm in proportional terms. Then

$$\mathrm{Var}(X_2) = \mathrm{Var}\ (X_1) + \mathrm{Var}(X_2-X_1) + 2\mathrm{Cov}(X_1,\ X_2-X_1)$$

or

$$\sigma^2_{X_2} = \sigma^2_{X_1} + \sigma^2_{X_2-X_1} + 2\ \sigma_{X_1, X_2-X_1} = \sigma^2_{X_1} + \sigma^2_{X_2-X_1} + 2\rho\sigma_{X_1}\sigma_{X_2-X_1}$$

where $\rho = \dfrac{\mathrm{Cov}(X_1,X_2-X_1)}{\left\{\mathrm{Var}(X_1)\mathrm{Var}(X_2-X_1)\right\}^{-1/2}}$ is the correlation of growth and initial size. Gibrat's Law of Proportionate Effect assumes no correlation between growth and initial size, ie. $\rho=0$ and as a result concentration will increase continuously. More generally, we know that when $\rho\geq 0$ then (as it has been shown, see p.2 for case b≥1), variance in firm sizes (and thus concentration) will grow idefinitely. For a **fixed** number of firms:

$$\sigma^2_{X_2} - \sigma^2_{X_1} = \sigma^2_{X_2-X_1} + 2\rho\sigma_{X_1}\sigma_{X_2-X_1}$$

For $\rho>0$, the change in concentration $\sigma^2_{X_2} - \sigma^2_{X_1}$ is positively related to the variance of (the logs) of firm size changes $\sigma^2_{X_2-X_1}$ and to the correlation of growth with initial size. However, while if $\rho < 0$ concentration might decrease, it is also true that the difference in concentration,

$\sigma^2_{x_2}-\sigma^2_{x_1}$, will either increase more or decrease less the greater the variance in firm size changes, $\sigma^2_{x_2-x_1}$ is, provided that $\sigma_{x_2-x_1}>2\rho\sigma_{x_1}$. Weiss goes on to argue that there is greater dispersion in firm size changes $(\sigma_{x_2-x_1})$ in industries of durables and semi-durables where style and model change is the prevalent form of competition: 'the existence of a stock of previously produced goods might well result in new automobile models or new edition of introductory texts, even in the presence of perfect price collusion'. **Increases in concentration are thus anticipated to be positively correlated with frequent style of model change** -it will be highest in industries with differentiated durable and semi-durable goods.

Birth and Death Process (Simon and Bonini, 1958)

Allow births into the lowest size class of the distribution. They allow Gibrat's Law to operate therefore above some MES of firm. Firms are assumed born into this smallest size class at a constant rate θ (the probability of entry).For firm sizes 'sufficiently' above MES the distribution is approximately Pareto with an inequality parameter α where

$$\alpha = \frac{1}{1-\theta}$$

where $\theta = g/G$, G= net growth of assets of all firms in an industry in the period, g= part of G due to new firms.

Serial Correlation Models (Ijiri and Simon)

In these models growth is serially correlated

$$E(g_{t+1}) = k(t)\sum_{r=1}^{t}\left\{g_r\ \beta^r\right\}$$

g_t is the rate of growth during the t time interval

E( ) is the expected value operator

k(t) is a function of time, the same for all firms

β is the fraction that determines how rapidly the influence of past growth drops out.